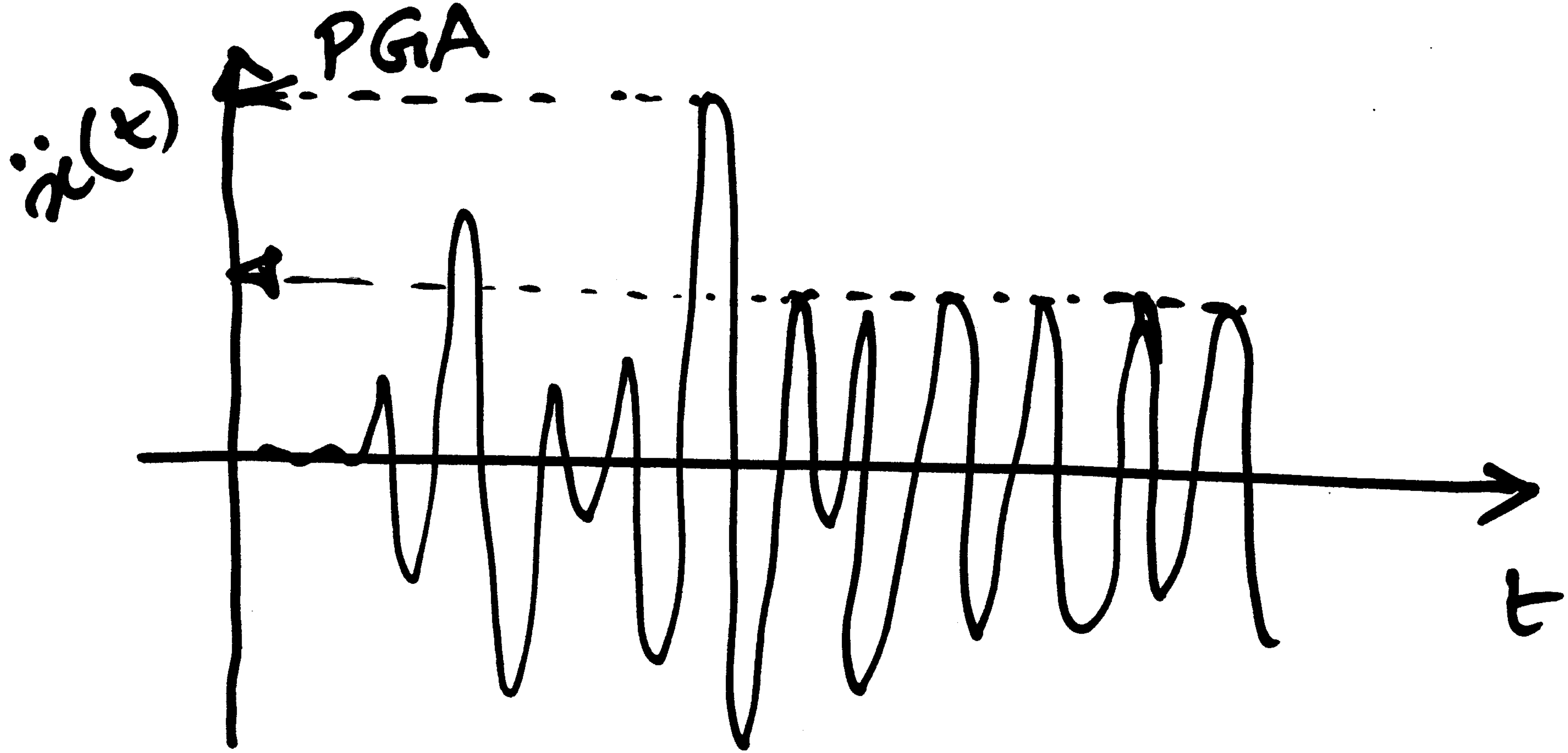
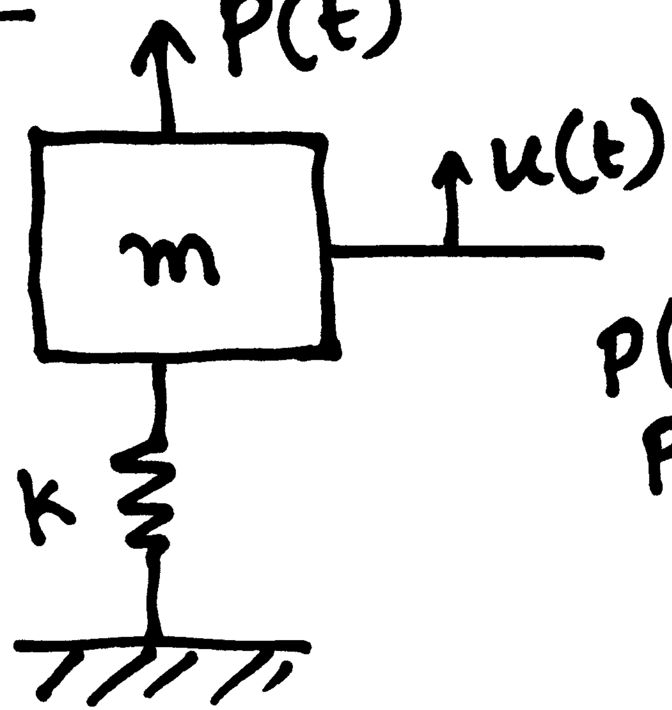


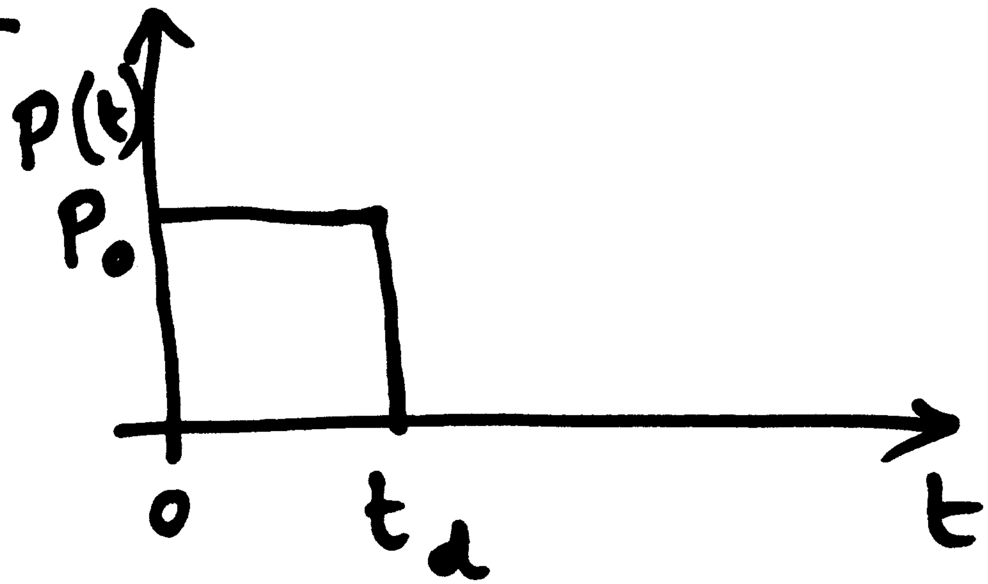
DC/0

SDOF

Lecture - 13

18/02/2013





$$u(t=0) = 0$$

$$\dot{u}(t=0) = 0$$

DC/1

(i) For Forced Vibration Phase
($t \leq t_d$)

$$\frac{u(t)}{u_{st}} = (1 - \cos \omega_n t)$$

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$= \left(1 - \cos \frac{2\pi t}{T_n}\right)$$

$$= \frac{2\pi}{T_n}$$

$$(0 \leq t \leq t_d)$$

(ii) For Free Vibration Phase

$(t \geq t_d)$

$$m\ddot{u} + ku = 0$$

$$u(t) = \left[u(t=t_d)\cos\omega_n(t-t_d) + \frac{\dot{u}(t=t_d)}{\omega_n}\sin\omega_n(t-t_d) \right]$$

At $t = t_d$, starting time for free vibration.

$$u(t=t_d) = \underline{u}_{st}\left(1-\cos\frac{2\pi t_d}{T_n}\right)$$

$$\dot{u}(t=t_d) = u_{st}\,\omega_n \sin\omega_n t_d$$

$$\frac{u(t)}{u_{st}} = \left(1-\cos\frac{2\pi t_d}{T_n}\right)\cos\omega_n(t-t_d)$$

$$+\sin\omega_n t_d \sin\omega_n(t-t_d)$$

$$\therefore \frac{u(t)}{u_{st}} = \left(2\sin\frac{\pi t_d}{T_n}\right)\sin\left[2\pi\left(\frac{t}{T_n}-\frac{1}{2}\cdot\frac{t_d}{T_n}\right)\right]$$

$$(t \geqslant t_d)$$

DC/4

$$\therefore \quad \frac{u(t)}{u_{st}} = f\left(\frac{t_d}{T_n}, \frac{t}{T_n}\right)$$

Maximum Response

(i) Forced Vibration Phase.

at $t = \frac{T_n}{2}$, $\left|\frac{u(t)}{u_{st}}\right|_{max} = 2{\cdot}0$

If, $t_d > \frac{T_n}{2}$,

$$R_d = \frac{u(t)}{u_{st}} = \begin{cases} 1 - \cos 2\pi \frac{t_d}{T_n} \; , & \frac{t_d}{T_n} \leq \frac{1}{2} \\ 2{\cdot}0 \; , & \frac{t_d}{T_n} \geq \frac{1}{2} \end{cases}$$

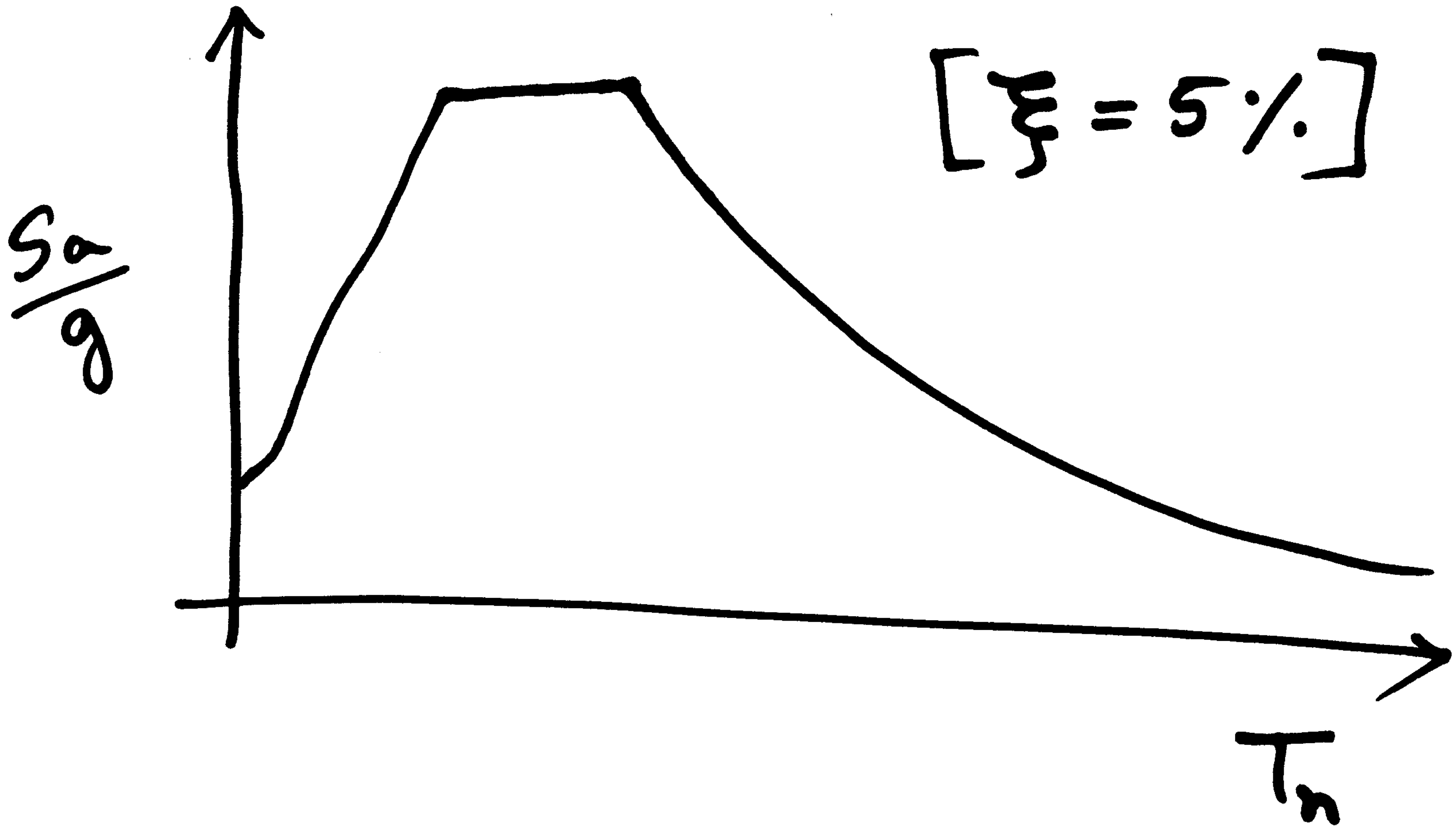
(ii) Free Vibration Phase

$$(u_t)_{max} = \sqrt{\left[u(t=t_d)\right]^2 + \left[\frac{\dot{u}(t=t_d)}{\omega_n}\right]^2}$$

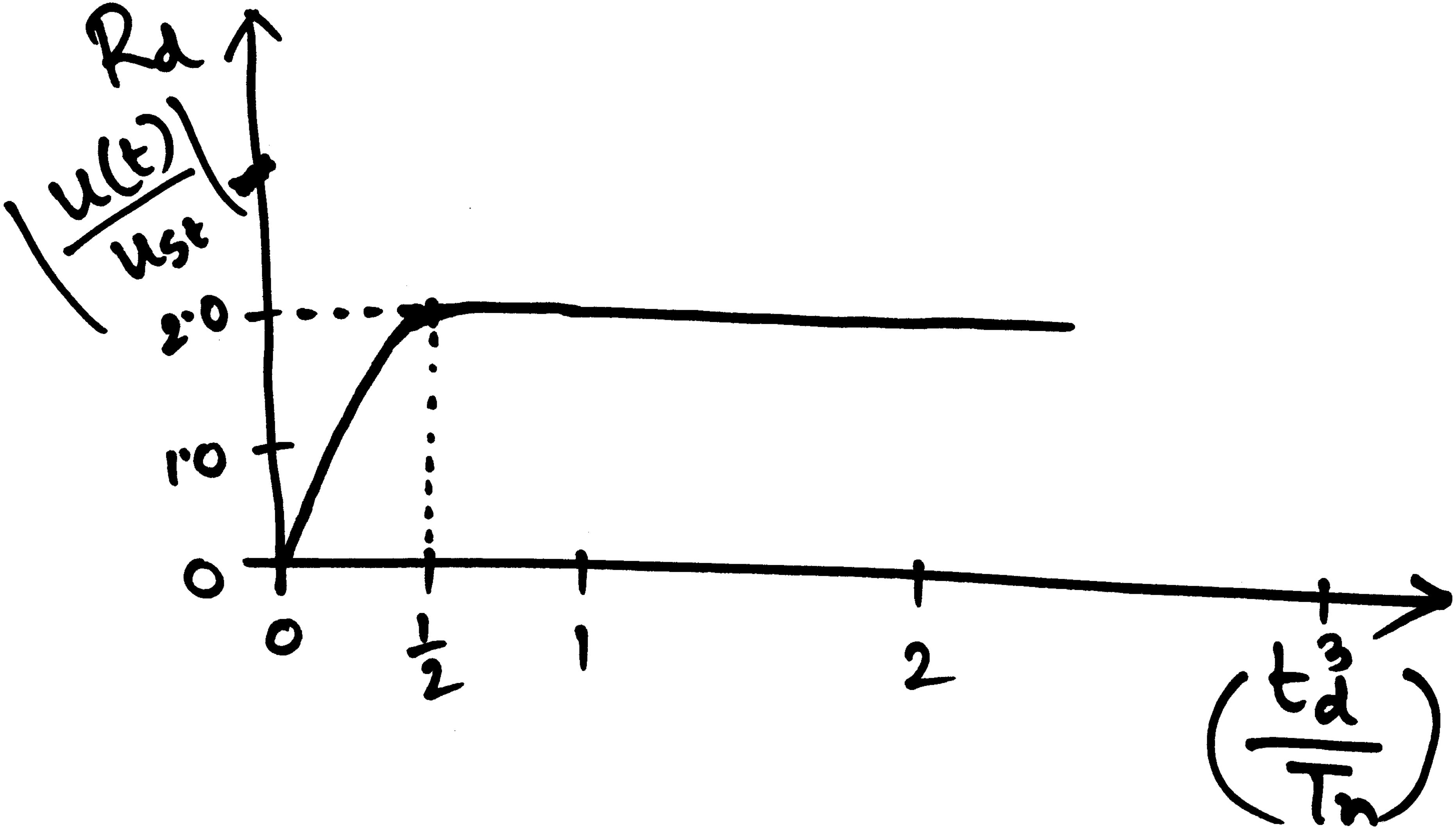
$$\left[u(t)\right]_{max} = 2(u_{st})\left|\sin \frac{\pi t_d}{T_n}\right|$$

$\therefore$ Absolute Maximum Response $(R_d) = 2\left|\sin\frac{\pi t_d}{T_n}\right|$

$$\therefore \quad R_d = \begin{cases} 2\left|\sin\frac{\pi t_d}{T_n}\right| & , \quad \frac{t_d}{T_n} \leq \frac{1}{2} \\ 2{\cdot}0 & , \quad \frac{t_d}{T_n} \geq \frac{1}{2} \end{cases}$$

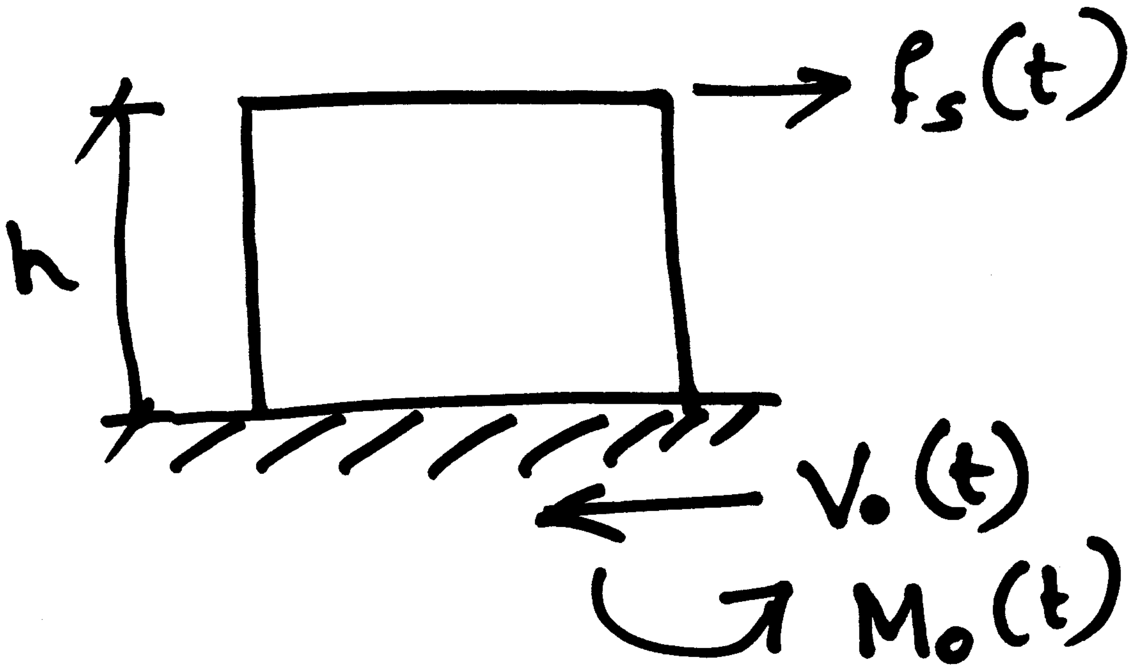


[ξ = 5%]
Sa/g
Tn



Rd
$\left(\frac{u(t)}{u_{st}}\right)$
2·0
1·0
0
0
$\frac{1}{2}$
1
2
3
$\left(\frac{t_d}{T_n}\right)$

DC/89



Denote $u_0 = \text{maximum } |u(t)|$

$$f_s(t) = k\,u(t)$$

$$= m\omega_n^2\,u(t)$$

$$= m \cdot A(t)$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$A(t) = \omega_n^2 u(t)$$

↑

Pseudo-acceleration.

Plot $u_o\left[|u(t)|_{max}\right]$ vs. T_n.

→ Displacement Response Spectrum

$\dot{u}_o = \text{maximum} |\dot{u}(t)|$ vs. T_n

$\rightarrow$ Velocity R.S.

$\ddot{u}_o = \text{maximum} |\ddot{u}(t)|$ vs. T_n

$\rightarrow$ Acceleration R.S.

Pseudo - velocity
Response Spectrum

$$\underset{(m/s)}{V} = \omega_n D = \frac{2\pi}{T_n} \cdot D$$

Pseudo - Acceleration
Response Spectrum.

$$\underset{(m/s^2)}{A} = \omega_n^2 \cdot D$$

$$u(t) = U \sin \omega t$$

$$|u(t)|_{max} = U$$

$$v_{max} = |\omega U|$$

$$a_{max} = |\omega^2 U|$$

$$\frac{v_{max}}{a_{max}} = \frac{1}{\omega} = \frac{T}{2\pi}$$

DC/14